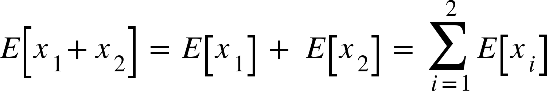
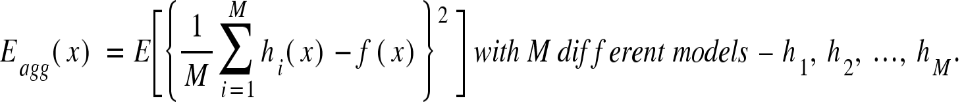
Nam Nguyen

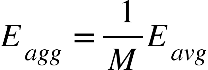
npn190000

1.

Consider 

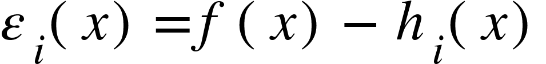
The error using the aggregated model is defined as:

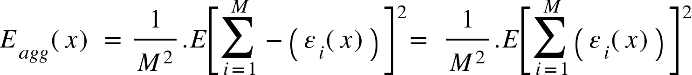


Prove: 

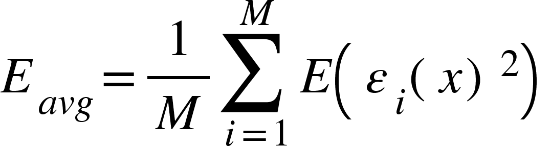
E subscript a g g end subscript open parentheses x close parentheses space equals space E open square brackets open curly brackets 1 over M sum from i equals 1 to M of h subscript i open parentheses x close parentheses minus f open parentheses x close parentheses close curly brackets squared space close square brackets space
space space space space space space space space space space space space space space space equals space E open square brackets 1 over M squared open square brackets sum from i equals 1 to M of h subscript i open parentheses x close parentheses minus f open parentheses x close parentheses close square brackets squared space close square brackets space
space space space space space space space space space space space space space space space equals space 1 over M squared. E open square brackets sum from i equals 1 to M of h subscript i open parentheses x close parentheses minus f open parentheses x close parentheses close square brackets squared space space
space space space space space space space space space space space space space space space equals space 1 over M squared. E open square brackets sum from i equals 1 to M of minus open parentheses f open parentheses x close parentheses minus h subscript i open parentheses x close parentheses close parentheses close square brackets squared space space

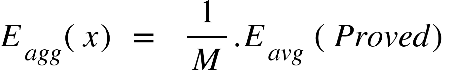
The error for each of the models would be described as:





The average value of the expected squared error for each of the models acting individually is defined as:



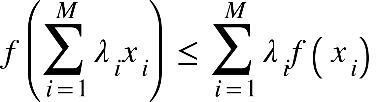


2.

In question 1, we had assumed that each of the errors are uncorrelated i.e. 𝐸 left parenthesis epsilon subscript i space left parenthesis 𝑥 right parenthesis epsilon subscript j left parenthesis 𝑥 right parenthesis right parenthesis space equals space 0 space f o r space a l l space i space not equal to space j

This is not really true, as the models are created using bootstrap samples and have correlation with each other. Now, let's remove that assumption. Show that using Jensen's inequality, it is still possible to prove that: E subscript a g g space end subscript less or equal than E subscript a v g end subscript

Jensen's inequality states that for any convex function f

E open square brackets x close square brackets space equals space sum from i equals 1 to M of lambda subscript i x subscript i space semicolon space space space E open square brackets f open parentheses x close parentheses close square brackets equals sum from i equals 1 to M of lambda subscript i f open parentheses x subscript i close parentheses


We have simple case convex function f: E open square brackets f open parentheses x close parentheses close square brackets equals lambda subscript 1 space space end subscript f open parentheses x subscript 1 close parentheses space plus lambda subscript 2 space space end subscript f open parentheses x subscript 2 close parentheses space greater or equal than f open parentheses lambda subscript 1 space space end subscript x subscript 1 space plus lambda subscript 2 space space end subscript x subscript 2 close parentheses equals space f open parentheses E open square brackets x close square brackets space close parentheses


Consider convex function f:

E open square brackets f open parentheses x close parentheses close square brackets equals sum from i equals 1 to M of lambda subscript i f open parentheses x subscript i close parentheses equals space open parentheses lambda subscript 1 space plus space lambda subscript 2 close parentheses open parentheses fraction numerator lambda subscript 1 space space end subscript f open parentheses x subscript 1 close parentheses space plus lambda subscript 2 space space end subscript f open parentheses x subscript 2 close parentheses space over denominator lambda subscript 1 space plus space lambda subscript 2 end fraction close parentheses plus space lambda subscript 3 space space end subscript f open parentheses x subscript 3 close parentheses plus... space plus lambda subscript M space space end subscript f open parentheses x subscript M close parentheses space

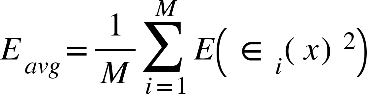

greater or equal than space open parentheses lambda subscript 1 space plus space lambda subscript 2 close parentheses times f open parentheses fraction numerator lambda subscript 1 space space end subscript x subscript 1 space plus lambda subscript 2 space space end subscript x subscript 2 space over denominator lambda subscript 1 space plus space lambda subscript 2 end fraction close parentheses plus space lambda subscript 3 space space end subscript f open parentheses x subscript 3 close parentheses plus... space plus lambda subscript M space space end subscript f open parentheses x subscript M close parentheses space

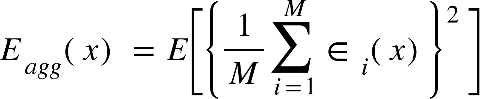

greater or equal than space f open parentheses open parentheses lambda subscript 1 space plus space lambda subscript 2 close parentheses times open parentheses fraction numerator lambda subscript 1 space space end subscript x subscript 1 space plus lambda subscript 2 space space end subscript x subscript 2 space over denominator lambda subscript 1 space plus space lambda subscript 2 end fraction close parentheses plus space lambda subscript 3 space space end subscript x subscript 3 plus... space plus lambda subscript M space space end subscript x subscript M space close parentheses


equals space f open parentheses lambda subscript 1 space space end subscript x subscript 1 space plus lambda subscript 2 space space end subscript x subscript 2 plus space lambda subscript 3 space space end subscript x subscript 3 plus... space plus lambda subscript M space space end subscript x subscript M space close parentheses equals space f open parentheses E open square brackets x close square brackets space close parentheses

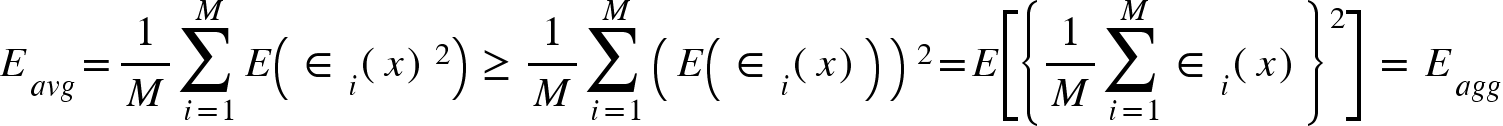

Hence: E[f(x)] ≥ f(E[x])

Applications of Jensen’s Inequality E(X2)≥ (E(X))2



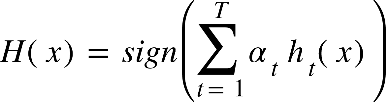


Consider: Jensen's inequality states that for any convex function f

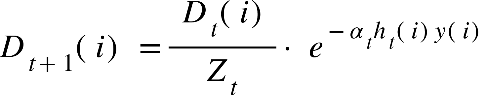


We can say that E subscript a g g space end subscript less or equal than E subscript a v g end subscript (proved)

3.



Also recall that the weight for the point i at step t+1 is given by:



Dt(i): ) is the normalized weight of point i in step t

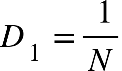
ℎ𝑡(𝑖) is the hypothesis (prediction) at step t for point i

𝛼𝑡 is the final “voting power” of hypothesis ℎ𝑡

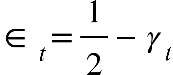
𝑦(𝑖) is the true label for point i

𝑍𝑡 is the normalization factor at step t (it ensures that the weights sum up to 1.0)

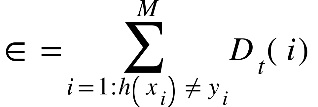
Note that at step 1, the points have equal weight



N is the total number of data points.

At each of the steps, the total error of ℎ𝑡 will be defined as 

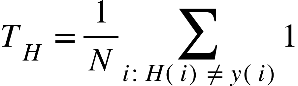
Error for Adaboost can be measure with respect to weight Dt .



While ht(xi) and yi both in {1;-1}

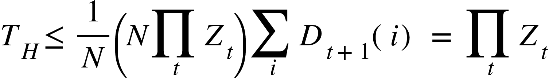
D subscript t plus 1 end subscript open parentheses i close parentheses space equals fraction numerator space D subscript 1 open parentheses i close parentheses over denominator Z subscript 1 end fraction times space e to the power of negative alpha subscript 1 h subscript 1 open parentheses i close parentheses y open parentheses i close parentheses end exponent times space fraction numerator space e to the power of negative alpha subscript 2 h subscript 2 open parentheses i close parentheses y open parentheses i close parentheses end exponent over denominator Z subscript 2 end fraction space... fraction numerator space space e to the power of negative alpha subscript i h subscript i open parentheses i close parentheses y open parentheses i close parentheses end exponent over denominator Z subscript i end fraction
equals fraction numerator space 1 over denominator N end fraction fraction numerator e to the power of sum from i equals 1 to t of minus alpha subscript i h subscript i open parentheses i close parentheses y open parentheses i close parentheses end exponent over denominator product from i equals 1 to t of Z subscript i end fraction equals fraction numerator space 1 over denominator N end fraction fraction numerator e to the power of negative f subscript t open parentheses i close parentheses y open parentheses i close parentheses end exponent over denominator product from i equals 1 to t of Z subscript i end fraction w i t h space f subscript t open parentheses i close parentheses space equals sum from i equals 1 to t of minus alpha subscript i h subscript i open parentheses i close parentheses
H e n c e colon space e to the power of negative f subscript t open parentheses i close parentheses y open parentheses i close parentheses end exponent equals N space open parentheses product from i equals 1 to t of Z subscript i close parentheses space sum for t of D subscript t plus 1 end subscript open parentheses i close parentheses
w i t h space sum for t of D subscript t plus 1 end subscript open parentheses i close parentheses equals 1

Now Total training error of H(x)



H open parentheses i close parentheses equals s i g n open parentheses f open parentheses i close parentheses close parenthesesso

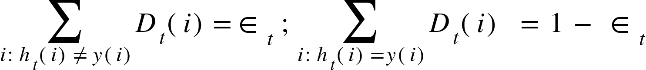
T subscript H space equals 1 over N sum from i colon space f open parentheses i close parentheses y open parentheses i close parentheses less or equal than 0 to blank of 1 less or equal than sum from i space to blank of e to the power of negative space f open parentheses i close parentheses y open parentheses i close parentheses end exponent space w h e n space e to the power of negative Z end exponent space greater or equal than 1 space w h e n space Z space less or equal than 0
H e n c e colon space T subscript H less or equal than sum from i space to blank of e to the power of negative space f open parentheses i close parentheses y open parentheses i close parentheses end exponent



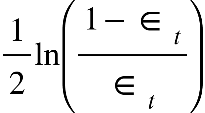
Now,

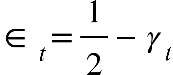
Z subscript t space equals sum for i of D subscript t open parentheses i close parentheses times space e to the power of negative alpha subscript t h subscript t open parentheses i close parentheses y open parentheses i close parentheses end exponent equals sum for i colon space h subscript t open parentheses i close parentheses equals y open parentheses i close parentheses of D subscript t open parentheses i close parentheses times space e to the power of negative alpha subscript t end exponent space plus sum for i colon space h subscript t open parentheses i close parentheses not equal to y open parentheses i close parentheses of D subscript t open parentheses i close parentheses times space e to the power of alpha subscript t end exponent

h subscript t open parentheses i close parentheses space equals space y open parentheses i close parentheses space t h e n space open curly brackets table row cell h subscript t open parentheses i close parentheses space equals space 1 end cell row cell y open parentheses i close parentheses space equals space 1 end cell end table close o r space open curly brackets table row cell h subscript t open parentheses i close parentheses space equals space minus 1 end cell row cell y open parentheses i close parentheses space equals space minus 1 end cell end table close
H e n c e colon space h subscript t open parentheses i close parentheses space times space y open parentheses i close parentheses equals 1
h subscript t open parentheses i close parentheses space not equal to space y open parentheses i close parentheses space t h e n space open curly brackets table row cell h subscript t open parentheses i close parentheses space equals space 1 end cell row cell y open parentheses i close parentheses space equals space minus 1 end cell end table close o r space open curly brackets table row cell h subscript t open parentheses i close parentheses space equals space 1 end cell row cell y open parentheses i close parentheses space equals space minus 1 end cell end table close
H e n c e colon space h subscript t open parentheses i close parentheses space times space y open parentheses i close parentheses equals negative 1

Now consider: 

Z subscript t space equals e to the power of negative alpha subscript t end exponent space open parentheses 1 space minus element of subscript t space close parentheses plus space e to the power of alpha subscript t end exponent space element of subscript t

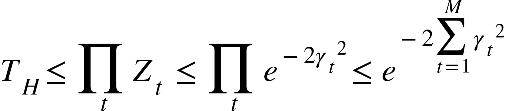
Prove that at the end of T steps, that mean minimize error TH with αt come to be 

Hence: Z subscript t space equals e to the power of negative alpha subscript t end exponent space open parentheses 1 space minus element of subscript t space close parentheses plus space e to the power of alpha subscript t end exponent space element of subscript t space equals space 2 square root of element of subscript t open parentheses 1 minus element of subscript t close parentheses end root with 

Z equals space square root of 1 minus 4 gamma subscript t end root

1 plus x space less or equal than e to the power of x space l e t space x space equals space minus 4 gamma subscript t squared
T h e n space 1 space minus space 4 gamma subscript t squared space less or equal than e to the power of negative 4 gamma subscript t squared end exponent
H e n c e colon space Z subscript t space space less or equal than space square root of e to the power of negative 4 gamma subscript t squared end exponent end root equals e to the power of negative 2 gamma subscript t squared end exponent


Consider:

(proved)